School-No: 003511

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IB Mathematics Analysis and Approaches
SL & HL
including German curriculum
(Abitur 2023)

Course Framework

- **The Math SL curriculum is combined with the national curriculum.**
- **The candidates are taught a 90-minute-class per week in special IB sessions; they are taught three to five 45-minute-classes per week in their German mathematics sessions**

Resources:

- IB course book SL: Mathematics: Analysis and Approaches SL. HODDER EDUCATION (2019)
- IB course book HL: Mathematics: Analysis and Approaches HIGHER LEVEL OXFORD (2019)
- German course book: Elemente der Mathematik Qualifikationsphase Nordrhein-Westfahlen Grundkurs or Leistungskurz (For topics that are not part of the IB curriculum and in addition to the coursebook)
- Old exam questions
- Questionbank
- Geogebra
- Excel
- Graphic Display Calculator TI Nspire CX

- **The IB Math SL-topics that are expanded on the national curriculum topics are marked in green.**
- **The IB Math SL-topics that are not part of the national curriculum at all are marked in blue.**
- **Questions relating theory of knowledge (TOK) and mathematics are marked in purple**
  Over the year the math- and the TOK-teacher constantly collaborate and harmonize the discussed topics.
- **The IB Math HL topics are marked in orange.**

Syllabus Content

The regular German math courses are taught according to our German school internal curriculum (see homepage). The additional, corresponding IB topics are taught with a time lag, following the German math lessons, due to the fact that these lessons are taught by several math colleagues and can therefore hardly be fully synchronized. This way the fundamental knowledge and techniques have already been laid.
In the column "German curriculum" only the IB relevant topics are specified. Other topics taught like linear geometry, matrices etc. can be found in the German internal school curriculum.

The given numbers in brackets refer to the syllabus. The chapters refer to the IB course book SL.

German curriculum	IB Math course curriculum
Pascal's triangle and ${}^{n}C_{r}$ (1.9)	**FIRST YEAR:** FIRST QUARTER **Sequences and Series (Chapter 2 and 13)** finite and infinite sequences and series, sigmanotation, recursive formula limits and convergence (1.2, 1.3 and 1.8) Financial applications of geometric sequences and series (compound interest, annual depreciation) (1.4) binomial theorem (1.9) Extension of the binomial theorem to fractional and negative indices, $(a+b)^n$, $n \in \mathbb{Q}$ (1.10) TOK: Is all knowledge concerned with identification and use of patterns? Consider Fibonacci numbers and connections with the golden ratio. TOK: How do mathematicians reconcile the fact that some conclusions seem to conflict with our intuitions? Consider for instance that a finite area can be bounded by an infinite perimeter. TOK: How have technological advances affected the nature and practice of mathematics? Consider the use of financial packages for instance. TOK: Is it possible to know about things of which we can have no experience, such as infinity? TOK: How have notable individuals shaped the development of mathematics as an area of knowledge? Consider Pascal and "his" triangle. **MOCK-EXAM I**

Proof (Chapter 11) Simple deductive proof, numerical and algebraic (1.6) **Functions (Chapter3, 4A, 14B, 15)** Straight line, gradient; intercepts, parallel and perpendicular lines (2.1) Determine key features of graphs, finding the points of intersection of two curves or lines using technology (2.4) Concept of a function, domain, range and graph. Function notation, concept that an inverse function (2.2-2.3, 2.5) quadratic functions in different forms (2.6) Odd and even functions. (2.14) Solution of quadratic equations The quadratic formula, The discriminant (2.7)	SECOND QUARTER **Proof** Proof by mathematical induction. Proof by contradiction. Use of a counterexample to show that a statement is not always true. (1.15) TOK: What is the difference between reasoning and mathematical proof? What is the difference between algebraic and geometric proof? Finding the inverse function, $f^{-1}(x)$, including domain restriction. Self-inverse functions. (2.14) **Functions (Chapter 14 A, 15B)** Composite functions (2.5) TOK: Do you think mathematics or logic should be classified as a language? TOK: Does studying the graph of a function contain the same level of mathematical rigour as studying the function algebraically? What are the advantages and disadvantages of having different forms and symbolic language in mathematics?

Functions (Chapter 16A) Transformations of graphs, Composite transformations (2.11) Polynomial functions, their graphs and equations; zeros, roots and factors. **Statistics (Chapter 6)** presentation of data: Histograms, frequency distributions, cumulative frequency, median, quartiles, percentiles, range and interquartile range (IQR), box and whisker diagrams (4.2) mean, median and mode, interquartile range, standard deviation and variance) (4.3)	THIRD QUARTER **Functions (Chapter 16B)** The reciprocal function, Rational functions of the form $f(x) = \frac{ax+}{cx+d}$ (2.8) **Rational functions of the form**: $f(x) = \frac{ax+b}{cx^2+dx+e}$ and $f(x) = \frac{ax^2+bx+c}{dx+e}$ (2.13) Partial fractions. (1.11) The graphs of functions $y = \|f(x)\|, y = f(\|x\|)$, $y = \frac{1}{f(x)}, y = f(ax+b), y = [f(x)]^2$ (2.16) Solution of quadratic inequalities. (2.7) Solution of modulus equations and inequalities. (2.16) Solutions of $g(x) \geq f(x)$, both graphically and analytically. (2.15) **Complex Numbers I** Complex numbers: number i, where $i^2 = -1$. Cartesian form $z = a + bi$; the terms real part, imaginary part, conjugate, modulus and argument. The complex plane. (1.12) The factor and remainder theorems. Sum and product of the roots of polynomial equations. (2.12) **Statistics (Chapter 6)** Concepts of population, sample, discrete and continuous data, reliability, outliers, sampling techniques (4.1) TOK: Why have mathematics and statistics sometimes been treated as separate subjects? How easy is it to be misled by statistics? Is it ever justifiable to purposely use statistics to mislead others?

	Statistics (Chapter 6 and 19 A) Effect of constant changes on the original data. Linear correlation of bivariate data. Pearson's product-moment correlation coefficient, r Scatter diagrams; lines of best fit, by eye, passing through the mean point. Regression line equation. Use of the equation for prediction purposes. (4.4 and 4.10) TOK: Correlation and causation–can we have knowledge of cause and effect relationships given that we can only observe correlation? What factors affect the reliability and validity of mathematical models in describing real-life phenomena? **MOCK-EXAM II**
Exponents and logarithms (Chapter 1 and 16 C) Operations with numbers in the form a × 10^k Laws of exponents with integer exponents. Introduction to logarithms with base 10 and e. Numerical evaluation of logarithms using technology. (1.5) Exponential functions in combination with polynomial functions and their graphs. (2.9) Modeling the world.	FOURTH QUARTER **Exploration** TOK: Do the names that we give things impact how we understand them? For instance, what is the impact of the fact that some large numbers are named, such as the googol and the googolplex, while others are represented in this form? TOK: Is mathematics invented or discovered? **Exponents and logarithms (Chapter 12, 16 C)** Laws of exponents with rational exponents. Laws of logarithms, change of base of a logarithm, solving exponential equations, including using logarithms. (1.7) Logarithmic functions and their graphs (2.9) TOK: What role do "models" play in mathematics? Do they play a different role in mathematics compared to their role in other areas of knowledge?

Vectors Concept of a vector; position vectors; displacement vectors. Representation of vectors using directed line segments. Base vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$. Components of a vector: $$\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v_1\boldsymbol{i} + v_2\boldsymbol{j} + v_3\boldsymbol{k}$$ Algebraic and geometric approaches to the following: • the sum and difference of two vectors • the zero vector **0**, the vector -**v** • multiplication by a scalar, *k***v**, parallel vectors • magnitude of a vector, $\|\boldsymbol{v}\|$; unit vectors, $\frac{\boldsymbol{v}}{\|\boldsymbol{v}\|}$ • position vectors $\overrightarrow{OA} = \boldsymbol{a}, \overrightarrow{OB} = \boldsymbol{b}$ • displacement vector $\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a}$ • Proofs of geometrical properties using vectors (3.12) The definition of the scalar product of two vectors. The angle between two vectors. Perpendicular vectors; parallel vectors. (3.13) Vector equation of a line in two and three dimensions: $$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$$ The angle between two lines Simple applications to kinematics. (3.14) Coincident, parallel, intersecting and skew lines, distinguishing between these cases. Points of intersection. (3.15) The definition of the vector product of two vectors. (3.16)	
	Properties of the vector product. Geometric interpretation of $\|v \times u\|$(3.16)
Vector equations of a plane: $$\mathrm{r} = \mathrm{a} + \lambda\mathrm{b} + \mu\mathrm{c}$$ where b and c are non-parallel vectors within the plane. $$\mathrm{r} \cdot \mathrm{n} = \mathrm{a} \cdot \mathrm{n}$$ where n is a normal to the plane and a is the position vector of a point on the plane. Cartesian equation of a plane $ax + by + cz = d$ (3.17) Intersections of: a line with a plane; two planes; three planes. Angle between: a line and a plane; two planes. (3.18)	

Equations (Chapter 17) Solving equations graphically. Use of technology (2.10) Solutions of systems of linear equations (a maximum of three equations in three unknowns), including cases where there is a unique solution, an infinite number of solutions or no solution. (1.16)	**<u>SECOND YEAR:</u>** <u>FIRST QUARTER</u> **Equations (Chapter 17)** Solving equations analytically. (2.10) TOK: What assumptions do mathematicians make when they apply mathematics to real-life situations?
Trigonometry (Chapter 4B, 5 and 18) The distance between two points in three-dimensional space, and their midpoint. Volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids. The size of an angle between two intersecting lines or between a line and a plane. (3.1) Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles. (3.2)	**Trigonometry (Chapter 5 and 18)** sine rule, cosine rule, Area of a triangle as $\frac{1}{2}ab\ sinC$ (3.2) Applications of right and non-right angled trigonometry, including Pythagoras's theorem. Angles of elevation and depression (3.3) The circle: radian measure of angles; length of an arc; area of a sector. (3.4) TOK: Which is a better measure of angle: radian or degree? What criteria can/do/should mathematicians use to make such decisions?
Definition of cosθ, sinθ in terms of the unit circle. (3.5)	Definition of $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$ Exact values of trigonometric ratios of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples. Extension of the sine rule to the ambiguous case. (3.5) The Pythagorean identity, Double angle identities for sine and cosine (3.6)
The circular functions sinx, cosx, and tanx; amplitude, their periodic nature, and their graphs Composite functions of the form f(x) = asin(b(x + c)) + d. (3.7)	Relationships between trigonometric functions and the symmetry properties of their graphs (3.11) TOK: Music can be expressed using mathematics. What does this tell us about the relationship between music and mathematics?

	Definition of the reciprocal trigonometric ratios $sec\theta, cosec\theta$ and $cot\theta$. Pythagorean identities: $1 + tan^2\theta = sec^2\theta$ $1 + cot^2\theta = cosec^2\theta$ The inverse functions $f(x) = arcsinx$, $f(x) = arccos, f(x) = arctan,$ their domains and ranges; their graphs (3.9) Compound angle identities. Double angle identity for tan. (3.10) Solving trigonometric equations in a finite interval, both graphically and analytically. Equations leading to quadratic equations in sinx, cosx or tanx. (3.8) **MOCK-EXAM III**
Calculus (Chapter 9, 20, 10, 21) Introduction to the concept of a limit. Derivative interpreted as gradient function and as rate of change. (5.1) Informal understanding of continuity and differentiability of a function at a point. Understanding of limits (convergence and divergence). Definition of derivative from first principles: $$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$ Higher derivatives. (5.12) Increasing and decreasing functions (5.2) The derivative of functions of the form $f(x) = ax^n + bx^{n-1} + \cdots$where all exponents are integers (5.3) Tangents at a given point, and their equations. (5.4) Normals at a given point, and their equations. (5.4) Derivative of x^n, e^x. Differentiation of a sum and a multiple of these functions. The chain rule for composite functions. The product rule (5.6) The second derivative. Graphical behaviour of functions, including the relationship between the graphs of $f, f', \text{ and } f''$ (5.7)	SECOND QUARTER TOK: What value does the knowledge of limits have? Is infinitesimal behaviour applicable to real life? Is intuition a valid way of knowing in mathematics? Derivative of x^n, sinx, cosx, e^x and lnx. Differentiation of a sum and a multiple of these functions. The quotient rules (5.6) TOK: What is the role of convention in mathematics? Is this similar or different to the role of convention in other areas of knowledge?

Local maximum and minimum points. Testing for maximum and minimum, Optimization, Points of inflexion with zero and non-zero gradients. (5.8)	The evaluation of limits of the form $\lim_{x \to a} \frac{f(x)}{g(x)}$ and $\lim_{x \to \infty} \frac{f(x)}{g(x)}$ using l'Hôpital's rule or the Maclaurin series. Repeated use of l'Hôpital's rule. (5.13) Implicit differentiation. Related rates of change. Optimisation problems. (5.14) Derivatives of tan*x*, sec*x*, cosec*x*, cot*x*, a^x,$\log_a x$, arcsin*x*, arccos*x*, arctan*x*. (5.15)
Introduction to integration as anti-differentiation, anti-differentiation with a boundary condition to determine the constant term, definite integrals using technology, Area of a region enclosed by a curve. (5.5) Indefinite integral of x^n (n ∈ ℚ) and e^x (5.10)	Indefinite integral of x^n (n ∈ ℚ), sinx, cosx, $\frac{1}{x}$ and e^x . The composites of any of these with the linear function ax + b. Integration by inspection (5.10) Indefinite integrals of the derivatives of any of the functions: tan*x*, sec*x*, cosec*x*, cot*x*, a^x,$\log_a x$, arcsin*x*, arccos*x*, arctan*x*. The composites of any of these with a linear function. Use of partial fractions to rearrange the integrand. (5.15) Integration by substitution. Integration by parts. Repeated integration by parts. (5.16)
Definite integrals, including analytical approach, Areas of a region enclosed by a curve y = f(x) and the x-axis, where f(x) can be positive or negative, without the use of technology. Areas between curves. (5.11) Area of the region enclosed by a curve and the y-axis in a given interval. Volumes of revolution about the x-axis or y-axis. (5.17)	TOK: An infinite area sweeps out a finite volume. Can this be reconciled with our intuition? Do emotion and intuition have a role in mathematics?

	Kinematic problems involving displacement s, velocity v, acceleration a and total distance travelled. (5.9) TOK: Is mathematics independent of culture? First order differential equations. Numerical solution of $\frac{dy}{dx} = f(x, y)$ using Euler's method. Variables separable. Homogeneous differential equation $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ using the substitution $y = vx$. Solution of $y' + P(x)y = Q(x)$, using the integrating factor. (5.18) Maclaurin series to obtain expansions for e^x, sin$x$, cos$x$, arctan$x$, ln$(1 + x)$,$(1 + x)^p$, $p \in \mathbb{Q}$. Use of simple substitution, products, integration and differentiation to obtain other series. Maclaurin series developed from differential equations.(5.19)
Probability (Chapter 7, 8, 19B, C) Concepts of trial, outcome, equally likely outcomes, relative frequency, sample space (U) and event, complementary events. Expected number of occurrences. (4.5) tree diagrams, sample space diagrams and tables of outcomes to calculate probabilities, Conditional probability, Independent events (4.6 and 4.11) Counting principles, including permutations and combinations. (1.10)	THIRD QUARTER **Complex Numbers II** Modulus–argument (polar) form: $z = r(cos\theta + isin\theta = rcis\theta)$ Euler form: $z = re^{i\theta}$ Sums, products and quotients in Cartesian, polar or Euler forms and their geometric interpretation. (1.13) Complex conjugate roots of quadratic and polynomial equations with real coefficients. De Moivre's theorem and its extension to rational exponents. Powers and roots of complex numbers. (1.14) **Probability (Chapter 7, 8, 19B, C)** TOK: To what extent are theoretical and experimental probabilities linked? What is the role of emotion in our perception of risk, for example in business, medicine and travel safety? Venn-Diagrams, Combined events: P(A ∪ B) = P(A) + P(B) − P(A ∩ B), Mutually exclusive events: P(A ∩ B) = 0 (4.6) TOK: Can calculation of gambling probabilities be considered an ethical application of mathematics? Should mathematicians be held responsible for unethical applications of their work? TOK: What do we mean by a "fair" game? Is it fair that casinos should make a profit?

Use of Bayes' theorem for a maximum of three events. (4.13) Concept of discrete random variables and their probability distributions. Expected value (mean), for discrete data (4.7) Binomial distribution. Mean and variance of the binomial distribution. (4.8)	Variance of a discrete random variable Continuous random variables and their probability density functions. Mode and median of continuous random variables. Mean, variance and standard deviation of both discrete and continuous random variables. The effect of linear transformations of X. (4.14) TOK: What criteria can we use to decide between different models? The normal distribution and curve, properties of the normal distribution, diagrammatic representation, normal probability calculations, inverse normal calculations (4.9) TOK: To what extent can we trust mathematical models such as the normal distribution? How can we know what to include, and what to exclude, in a model? Standardization of normal variables (z- values), Inverse normal calculations where mean or standard deviation are unknown. (4.12)
	FOURTH QUARTER Remaining Questions and Exam exercise

Learner Profils assignment to the aims

In the following the aims of the course are linked with the IB learner profile

The aims of all DP mathematics courses are to enable students to:

1. develop a curiosity and enjoyment of mathematics, and appreciate its elegance and power (INQUIRERS)

2. develop an understanding of the concepts, principles and nature of mathematics (KNOWLEDGEABLE)

3. communicate mathematics clearly, concisely and confidently in a variety of contexts (COMMUNICATORS)

4. develop logical and creative thinking, and patience and persistence in problem solving to instill confidence in using mathematics (THINKERS)

5. employ and refine their powers of abstraction and generalization (KNOWLEDGEABLE)

6. take action to apply and transfer skills to alternative situations, to other areas of knowledge and to future developments in their local and global communities (OPEN-MINDED)

7. appreciate how developments in technology and mathematics influence each other

8. appreciate the moral, social and ethical questions arising from the work of mathematicians and the applications of mathematics (CARING)

9. appreciate the universality of mathematics and its multicultural, international and historical perspectives (OPEN-MINDED)

10. appreciate the contribution of mathematics to other disciplines, and as a particular “area of knowledge” in the TOK course (OPEN-MINDED)

11. develop the ability to reflect critically upon their own work and the work of others (REFLECTIVE)

12. independently and collaboratively extend their understanding of mathematics (KNOWLEDGEABLE)

[Aims are quoted from: Mathematics: analysis and approaches guide First assessment 2021 Ed. by the International Baccalaureate Organisation, 2019]

Assessment objectives

Problem solving is central to learning mathematics and involves the acquisition of mathematical skills and concepts in a wide range of situations, including non-routine, open-ended and real-world problems. Having followed a DP mathematics course, students will be expected to demonstrate the following:

Assessment objectives	Paper 1 %	Paper 2 %	Paper 3 % HL only	Exploration %
Knowledge and understanding	20-30	15-25	10-20	5-15
Problem solving	20-30	15-25	20-30	5-20
Communication and interpretation	20-30	15-25	15-25	15-25
Technology	0	25-35	10-30	10-20
Reasoning	5-15	5-10	10-20	5-25
Inquiry approaches	10-20	5-10	15-30	25-35

1. Knowledge and understanding: Recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of familiar and unfamiliar contexts.
2. Problem solving: Recall, select and use their knowledge of mathematical skills, results and models in both abstract and real-world contexts to solve problems.
3. Communication and interpretation: Transform common realistic contexts into mathematics; comment on the context; sketch or draw mathematical diagrams, graphs or constructions both on paper and using technology; record methods, solutions and conclusions using standardized notation; use appropriate notation and terminology.
4. Technology: Use technology accurately, appropriately and efficiently both to explore new ideas and to solve problems.
5. Reasoning: Construct mathematical arguments through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions.
6. Inquiry approaches: Investigate unfamiliar situations, both abstract and from the real world, involving organizing and analyzing information, making conjectures, drawing conclusions, and testing their validity.

SL:

First assessment 2021

Assessment component	Weighting
External assessment (3 hours)	**80%**
Paper 1 (90 minutes) No technology allowed. (80 marks) *Section A* Compulsory short-response questions based on the syllabus. *Section B* Compulsory extended-response questions based on the syllabus.	**40%**
Paper 2 (90 minutes) Technology required. (80 marks) *Section A* Compulsory short-response questions based on the syllabus. *Section B* Compulsory extended-response questions based on the syllabus	**40%**
Internal assessment This component is internally assessed by the teacher and externally moderated by the IB at the end of the course. **Mathematical exploration** Internal assessment in mathematics is an individual exploration. This is a piece of written work that involves investigating an area of mathematics. (20 marks)	**20%**

HL:

First assessment 2021

Assessment component	Weighting
External assessment (5 hours)	**80%**
Paper 1 (120 minutes) No technology allowed. (110 marks) *Section A* Compulsory short-response questions based on the syllabus. *Section B* Compulsory extended-response questions based on the syllabus.	**30%**
Paper 2 (120 minutes) Technology required. (110 marks) *Section A* Compulsory short-response questions based on the syllabus. *Section B* Compulsory extended-response questions based on the syllabus.	**30%**
Paper 3 (60 minutes) Technology required. (55 marks) Two compulsory extended response problem-solving questions.	**20%**
Internal assessment This component is internally assessed by the teacher and externally moderated by the IB at the end of the course. **Mathematical exploration** Internal assessment in mathematics is an individual exploration. This is a piece of written work that involves investigating an area of mathematics. (20 marks)	**20%**

[Mathematics: analysis and approaches guide First assessment 2021 Ed. by the International Baccalaureate Organisation, 2019]

Exploration:

In the beginning of the first schoolyear the students are introduced to the concept of the mathematical exploration and are asked to watch out for areas of mathematics or concepts they would like to explore. In the third quarter of the first schoolyear the statistic topic including the linear regression is taught, because a lot of explorations are traditionally based on this topic. After that (approximately 3 weeks before easter break) the students are introduced to the details and assessment criteria of the exploration. They use the criteria to assess an example from the support material. Examples of promising and inappropriate topics as well as topic finding methods are discussed. After that the students have one month to do some research and declare their focus. After another month (about end of May) the first draft has to be handed over to the teacher. After getting feedback, the students have 10 days to finish their final draft before summer break.

Mathematics and international-mindedness

International-mindedness should be taught by showing that…

- many foundations of modern mathematics were laid by diverse cultures
- mathematics can be seen as an international language – ready to communicate with people from around the world
- mathematics is an important source throughout history for many worldwide inventions and the development of architecture, trade and navigation.
- mathematicians today and in the past are important members in every governmental structure and have influence on important political decisions (for example fighting worldwide pandemics or prohibit climate change)

Especially when finding a topic for the exploration the teacher encourages the students to widen their horizon writing the exploration in relation to the aspects named above.

The students are encouraged to participate in different Erasmus-projects connecting to different subjects such as mathematics. Meeting students out of other counties gives opportunities to get to know other international perspectives.

Mathematics and CAS

Acquired mathematical knowledge and skills through the IB-program can be successfully used to develop, plan and deliver CAS-Projects or help other students who are struggling with mathematic problems in the following ways:

- internal student coaching
- support the evaluation of international mathematical competitions (Kängurutest etc.)
- planning and calculating the costs of the graduation ball
- support student council creating and evaluating internal student surveys
- planning events like the $\pi - day$ to raise money for social projects

Approach to learning – Teaching unit

Mathematical Modelling with exponential functions
(Topic and teaching unit fosters different skills and matches many of the faces of the IB Learner Profile)

Students

- conduct an experiment – they measure the temperature of coffee in a cup in groups
- gain and present data in an appropriate way
- present diagrams, discuss measurement faults and show use of technology
- discuss characteristics of the data and compare different measurements concerning different test parameters (starting temperature, thickness of the cup, etc.)
- search for appropriate functions in the sense of best approximation by using the regression functions of the GDC (or other tools) e.g. polynomial functions of grade 3 or 4.
- clarify that these functions do fit the data quite well, but are no useful models for describing the physical process when used for future prognosis (coffee cannot freeze, coffee cannot get hotter, etc.)
- discuss possible meanings of „model“ in mathematics
- use the cognitive conflict and develop a new model which fits to the data and the physical problem (Newtons law of cooling is discovered)
- use exponential functions/combined functions
- do some exercises around the topic „cooling processes“ in different settings (groupwork)
- explain and discuss the results
- look for further applications in real life situations and try to expand the model by contacting experts (e.g. cooling of corpses in cooperation with the forensic medicine)